Fig. 6. Current distribution on two symmetrically located strips.  $\lambda/a = 1.591$ .

## V. CONCLUSIONS

We have illustrated the applicability of the SDFEM to a specific set of rectangular waveguide discontinuity problems. The numerical properties of the solution were studied and its accuracy verified. Further work in this area should now be done to apply the SDFEM to problems with thick and/or cascaded discontinuities in waveguides of arbitrary cross-section. In these cases the general  $TM_{mn}$  and  $TE_{mn}$  modes should be included in the analysis.

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## Accurate and Efficient Computation of Dielectric Losses in Multi-Level, Multi-Conductor Microstrip For CAD Applications

James P. K. Gilb and Constantine A. Balanis

**Abstract**—Accurate and efficient computation of dielectric losses in complex microstrip structures is important in the computer-aided design of microwave and millimeter-wave integrated circuits. The proposed approach can be used in lieu of lossy, full-wave solutions to provide accurate and efficient data for the CAD of multi-level, multi-conductor MIC and MMIC structures. This new application gives results that are as accurate as lossy full-wave techniques over a wide range of frequencies, including the dispersive region. In addition to providing accurate results, this method is up to three times faster, depending on the number and type of substrates or superstrates. Results are shown for various multi-conductor, multi-level structures which compare well with the lossy, full-wave approach and require significantly less computer time.

## I. INTRODUCTION

One of the most important goals in the computer modeling of MIC's and MMIC's is to provide highly accurate simulations in order to reduce the number of design iterations. Accurate modeling of all of the characteristics of multi-level, multi-conductor structures is necessary in the quest for single iteration design of complex circuits. At the same time, to facilitate the design process, these accurate methods must also provide results as quickly as possible. Current techniques available for the calculation of the dielectric attenuation coefficient compromise on either accuracy or speed, and many are not suitable for complex structures. In addition, lossy full-wave techniques usually require completely different subroutines and utility libraries. A new application of an old formulation is presented here which provides accurate results for the dielectric loss coefficient for multi-level, multi-conductor structures in about one-third the time required for a lossy, full-wave computation. This new approach is ideally suited for CAD applications since it uses currently available lossless techniques and does not require special subroutines and software libraries.

Various full-wave methods have been used to compute the dielectric loss in multi-layer, multi-conductor structures. The Spectral Domain Approach (SDA) has been used, both with a perturbational formula for the attenuation coefficient [1] and by formulating the problem with a complex dielectric constant [2]-[4]. Other full-wave techniques that have been used include the space-domain, moment method [5], and the Finite-Difference Time-Domain (FDTD) [6]. All of these techniques give accurate results for the dielectric loss in a general microstrip structure, but they require a significant amount of computational effort. An alternate approach is to use an approximate formula for the dielectric loss coefficient. One of the most widely used formulas for computing the dielectric attenuation coefficient is the one advanced by Schneider [7]. This formula has long been used with approximate formulas for  $\epsilon_{\text{eff}}$  to compute the dielectric attenuation coefficient,  $\alpha_d$ . It was recently shown that this formula gives results that are as accurate as those obtained with a lossy

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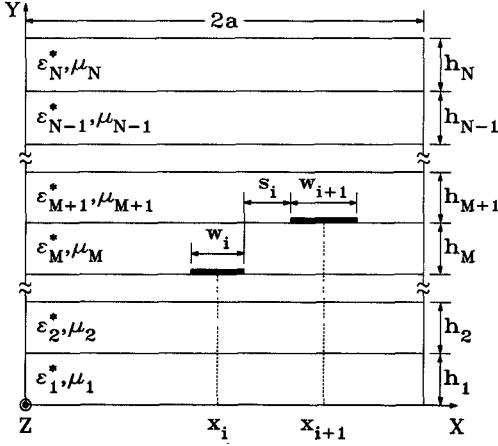


Fig. 1. The geometry of an asymmetric multi-level, multi-conductor interconnect.

full-wave approach, if the partial derivative of  $\epsilon_{\text{eff}}$  is computed accurately [4]. In addition, since the derivation in [7] was done in a general manner, the formula is valid for multi-level, multi-conductor structures, provided that the partial derivative of the modal effective dielectric constant with respect to the dielectric constant of the substrate layer can be computed.

Although this formula has been verified for single-substrate, single-conductor microstrip structures, there has not yet been a study to determine if this formula can be successfully applied to more general structures. A lossless Spectral Domain Approach (SDA) is used here in conjunction with a finite difference approximation of the derivative to accurately compute losses in microstrip lines using Schneider's formula. To verify the accuracy of the formula, these results are compared to data obtained with a lossy SDA formulation. The numerical efficiency of the method is analyzed for multi-layer structures where some or all of the layers are lossy. In addition, it is shown that the formula may also be used to accurately compute the modal dielectric attenuation coefficients in a multi-conductor structure. These results show that using Schneider's formulation is faster than using a full-wave, lossy formulation while giving results that are just as accurate.

## II. ANALYSIS

A general, lossy, multi-level, multi-conductor microstrip line is considered, as shown in Fig. 1. The structure is surrounded on all four sides by perfect electric conductors at  $x = 0, x = 2a, y = 0$ , and  $y = \sum_{i=1}^N h_i$ . For an open structure,  $a \rightarrow \infty$  and  $h_N \rightarrow \infty$  whereas for a covered structure without sidewalls,  $a \rightarrow \infty$  while  $h_N$  remains finite. There may be any number of conductors located on any of the dielectric interfaces. Two conductors are shown in Fig. 1 located at  $x = x_i, y = \sum_{i=1}^M h_i$  and  $x = x_{i+1}, y = \sum_{i=1}^M h_i$  with widths  $w_i + w_{i+1}$ , respectively. The conductors are separated by a spacing  $s_i$ , which is always measured in the  $x$  direction between the near edges of adjacent conductors. The complex effective relative dielectric constant is defined as

$$\epsilon_{\text{eff}}^* = \epsilon'_{\text{eff}} - j\epsilon''_{\text{eff}} = \frac{\gamma_z^2}{\omega^2 \mu_0 \epsilon_0} \quad (1)$$

where  $\gamma_z$  is the complex propagation constant of the structure.

The formula given by Schneider for the effective loss tangent for this structure is given by [7]

$$(\tan \delta)_{\text{eff}} = \frac{\epsilon''_{\text{eff}}}{\epsilon'_{\text{eff}}} = \frac{1}{\epsilon_{\text{eff}}} \sum_{n=1}^N \epsilon_{rn} \frac{\partial \epsilon_{\text{eff}}}{\partial \epsilon_{rn}} \tan \delta_n \quad (2)$$

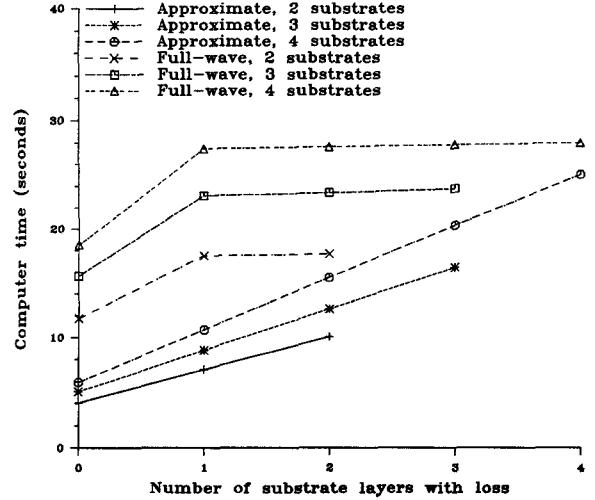


Fig. 2. Computation times for various substrate configurations using (3) and the lossy SDA.

where  $\epsilon_{rn}$  and  $\tan \delta_n$  are the relative dielectric constants and loss tangents, respectively, of the  $n$ th dielectric substrate and  $N$  is the total numbers of dielectric layers. In the above formula  $\epsilon_{\text{eff}}$  (without primes) represents the effective relative dielectric constant of the equivalent lossless problem. Although the formula was derived assuming a quasi-TEM mode, which is valid for microstrips at lower frequencies, this formula can be used at higher frequencies by including the frequency dependence for  $\epsilon_{\text{eff}}$  in the formula and partial derivatives.

The derivation given in [7] is valid for any mode which can be excited in a given structure. Thus, an effective loss tangent can be defined for each of the  $M$  independent modes in an  $M$ -conductor structure as

$$(\tan \delta)_{\text{eff}(i)} = \frac{\epsilon''_{\text{eff}(i)}}{\epsilon'_{\text{eff}(i)}} = \frac{1}{\epsilon_{\text{eff}(i)}} \sum_{n=1}^N \epsilon_{rn} \frac{\partial \epsilon_{\text{eff}(i)}}{\partial \epsilon_{rn}} \tan \delta_n \quad (3)$$

where  $\epsilon_{\text{eff}(i)}^* = \epsilon'_{\text{eff}(i)} - j\epsilon''_{\text{eff}(i)}$  is the complex relative effective dielectric constant of the  $i$ th mode and  $\epsilon_{\text{eff}(i)}$  is the effective relative dielectric constant of the  $i$ th mode for the equivalent lossless problem. Again, this formula can also be used at higher frequencies by including a frequency dependence for  $\epsilon_{\text{eff}(i)}$  and its partial derivatives.

For low-loss structures, it has been shown that (3) will give good results if the partial derivative is computed accurately. Unfortunately, closed-form expressions for  $\epsilon_{\text{eff}}$  are not available for a wide variety of multi-level and/or multi-conductor structures. An alternative is to compute the partial derivative using a finite difference approximation [4] with  $\epsilon_{\text{eff}}$  determined by full-wave, lossless method, such as the SDA. In this paper, the SDA, as described in [8], is used to compute the lossless  $\epsilon_{\text{eff}}$  used in (3) and in the partial derivatives. Although the SDA is used in this paper, any other accurate, lossless formulation can be used with (3) and the finite difference derivative to compute the dielectric attenuation coefficient.

## III. RESULTS

Since (3) can be used for a wide variety of microwave circuit designs, its numerical efficiency is an important consideration. In Fig. 2, the time required to compute the effective dielectric constant and dielectric attenuation coefficient, for three structures with multiple substrates, is presented as a function of the number of substrate layers that are lossy. A single center conductor, open microstrip structure is used with  $w = 0.6$  mm. The computation times are for an IBM

TABLE I  
SUBSTRATE PARAMETERS FOR 2-SUBSTRATE STRUCTURE  
( $h_1 = 0.335$  mm,  $h_2 = 0.3$  mm,  $\epsilon_{r1} = 2.2$ ,  $\epsilon_{r2} = 9.7$ )

# of lossy layers	$\tan \delta_1$	$\tan \delta_2$
0	0.0	0.0
1	0.0	0.0001
2	0.0001	0.0001

TABLE II  
SUBSTRATE PARAMETERS FOR 3-SUBSTRATE STRUCTURE ( $h_1 = 0.335$  mm,  $h_2 = 0.3$  mm,  $h_3 = 0.3$  mm,  $\epsilon_{r1} = 2.2$ ,  $\epsilon_{r2} = 4.4$ ,  $\epsilon_{r3} = 9.7$ )

# of lossy layers	$\tan \delta_1$	$\tan \delta_2$	$\tan \delta_3$
0	0.0	0.0	0.0
1	0.0001	0.0	0.0
2	0.0001	0.0	0.0001
3	0.0001	0.0020	0.0001

TABLE III  
SUBSTRATE PARAMETERS FOR 4-SUBSTRATE STRUCTURE  
( $h_1 = 0.2$  mm,  $h_2 = 0.2$  mm,  $h_3 = 0.2$  mm,  $h_4 = 0.2$  mm,  $\epsilon_{r1} = 2.2$ ,  $\epsilon_{r2} = 4.4$ ,  $\epsilon_{r3} = 9.7$ ,  $\epsilon_{r4} = 6.8$ )

# of lossy layers	$\tan \delta_1$	$\tan \delta_2$	$\tan \delta_3$	$\tan \delta_4$
0	0.0	0.0	0.0	0.0
1	0.0	0.0	0.0	0.0001
2	0.0	0.0	0.0002	0.0001
3	0.0	0.0001	0.0002	0.0001
4	0.0020	0.0001	0.0002	0.0001

3090 using the same number of spectral components for the lossy and lossless cases and computing 81 values of  $\epsilon_{\text{ref}}^*$  for each mode. Throughout the frequency range, the results using (3) agreed with the lossy SDA results to within the accuracy of the full-wave method. Tables I-III summarize the substrate parameters used for Fig. 2.

If none of the substrate layers is lossy, then (3) is two to three times faster than the full-wave approach, as shown in Fig. 2. As the number of lossy layers increases, the computation time for the approximate formula increases linearly, since one additional partial derivative must be computed for each additional lossy layer. On the other hand, for additional lossy layers, the full-wave method requires very little additional effort to compute the complex propagation constant. Thus, the execution times of the full-wave method are fairly constant as the number of lossy substrate layers increases. However, for all cases, using (3) gives results that are just as accurate as the full-wave method, using less computer time. Using (3) with the lossless  $\epsilon_{\text{ref}}$  gives accurate results as long as the presence of losses do not significantly affect the field structure, usually if  $(\tan \delta_i)_{\text{max}} < 0.1$ .

The characteristics of lossy, asymmetric coupled microstrips are shown in Fig. 3 for the  $c$  and  $\pi$  modes as a function of frequency. The maximum disagreement for  $\epsilon_{\text{ref}}''$  of both modes between the lossy SDA and (3) was 0.24% and the average disagreement was 0.046%. The time required to compute  $\epsilon_{\text{ref}}^*$  for both modes was 39.06 seconds for the lossy SDA and only 23.91 seconds using (3). For single layer structures, both  $\epsilon_{\text{ref}}'$  and  $\epsilon_{\text{ref}}''$  for the  $c$  mode are greater than those of the  $\pi$  mode. However, due to the choice of the substrate configuration and electrical parameters, in Fig. 3,  $\epsilon_{\text{ref}}'$  for the  $\pi$  mode is greater than that of the  $c$  mode while  $\epsilon_{\text{ref}}''$  for the  $c$  mode is greater than that of the  $\pi$  mode.

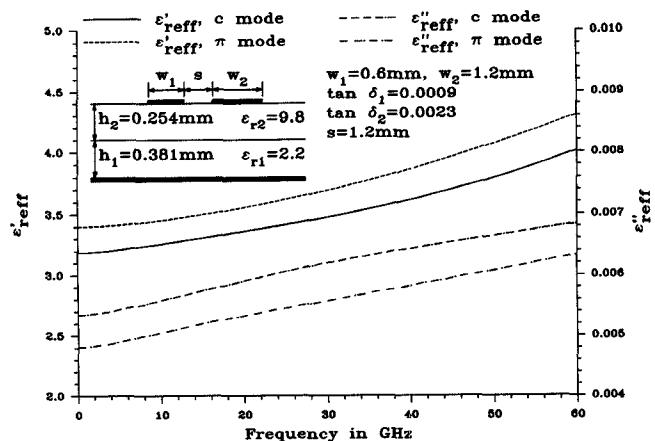


Fig. 3.  $\epsilon_{\text{ref}}'$  and  $\epsilon_{\text{ref}}''$  of the  $c$  and  $\pi$  modes of a two-substrate, asymmetric coupled open microstrips as function of frequency.

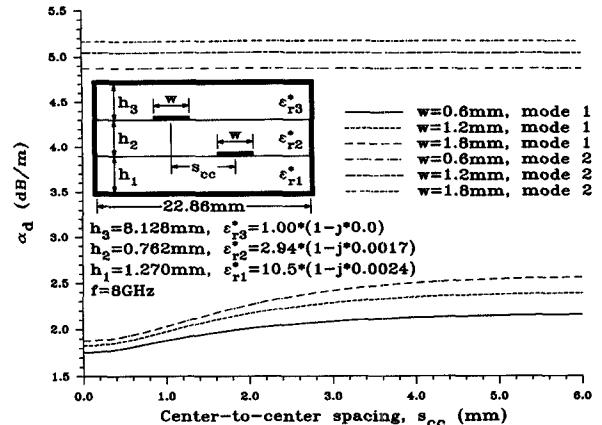


Fig. 4. Dielectric attenuation coefficient,  $\alpha_d$ , as a function of the center-to-center spacing of the center conductors for multi-level microstrips in X-band waveguide.

An example of a multi-level, multi-conductor structure is analyzed in Fig. 4, where a two-layer substrate with two center conductors on different levels is placed in an  $X$ -band waveguide. Three center conductor widths are considered and the dielectric loss coefficient is plotted as a function of the center-to-center spacing,  $s_{\text{cc}}$ , for the two independent modes. When the center-to-center spacing is zero the conductors are completely overlapped as in broadside coupled lines. Since this structure does not possess a line of symmetry, the independent modes are neither even nor odd modes. For large spacings,  $s_{\text{cc}} > 1.0$ , mode 2 can be characterized as a  $c$ , or in-phase mode, and mode 1 as a  $\pi$ , or anti-phase mode, as is traditional for asymmetric coupled lines. However, for small  $s_{\text{cc}}$  both modes are orthogonal, anti-phase modes so  $c$  and  $\pi$  designations are not appropriate. The maximum difference in the results for  $\alpha_d$  using the lossy SDA and (3) was only 0.30% for the two modes and all three center conductor widths while the average disagreement between the two methods was 0.12%. The results for 60 values of  $\epsilon_{\text{ref}}^*$  for each of the six graphs required 90.64 seconds using the lossy SDA and only 58.63 seconds using (3).

For both modes, the dielectric attenuation coefficient increases with increasing center conductor width, since the fields are more confined in the lossy substrates than in the air. As the spacing increases,  $\alpha_d$ , for mode 1 increases, approaching the value of an isolated conductor on a two-layer substrate. On the other hand, the  $\alpha_d$  for mode 2 increases

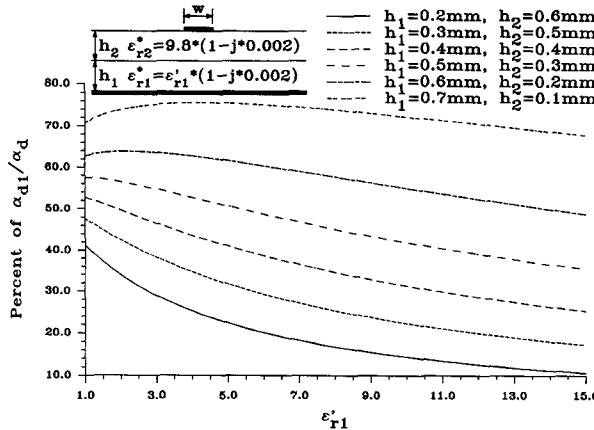


Fig. 5. Percent contribution of  $\alpha_{d1}$  to the total dielectric attenuation coefficient,  $\alpha_d$ , as a function of the dielectric constant of the lower substrate for an open microstrip ( $f = 1$  GHz,  $w = 0.6$  mm).

only slightly with increasing spacing and then decreases very slightly to approach the value of an isolated conductor embedded between two dielectric layers. The reason that  $\alpha_d$  changes so slightly for mode 2 is that it is either in-phase or only slightly anti-phase for all  $s_{cc}$ , and so the field lines are confined mostly within the two substrate layers for all spacings. On the other hand, for small spacings most of the field lines for mode 1 are between the two conductors, while for large spacings more of the field lines are distributed in the air and the lower substrate. The  $\epsilon'_{\text{eff}}$ 's of the two modes show behavior similar to that of the corresponding  $\alpha_d$ 's.

An additional benefit in using (3) instead of a lossy, full-wave method is that it can isolate the contribution of each dielectric layer to the attenuation coefficient. This can be useful in the design process because it allows the designer to see how each dielectric layer affects the dielectric attenuation coefficient. To get this information using a full-wave, lossless technique, the entire problem must be solved once for each of the layers with loss, with all the other layers being lossless. On the other hand, (3) determines the attenuation coefficient for each of the dielectric layers and adds their values to find the total dielectric loss coefficient. Thus, using (3) requires no extra time to determine the contribution of each layer to the total attenuation coefficient, making it much more efficient for these types of computations.

In Fig. 5, a two-substrate, single center conductor open microstrip is examined to illustrate how changing the dielectric constant of the lower substrate affects the contribution of a particular layer to the total dielectric loss. Six different combinations of  $h_1$  and  $h_2$  are considered and the percentage of  $\alpha_{d1}/\alpha_d$  is plotted as a function of the relative dielectric constant of the lower substrate,  $\epsilon_{r1}$ . The maximum disagreement between the lossy SDA and (3) was 0.66% for all four modes and the average disagreement was 0.18%. The lossy SDA required 138.49 seconds to compute 60 percentages of  $\alpha_{d1}/\alpha_d$  for each of the six substrate combinations while (3) used only 60.74 seconds. When the lower substrate is relatively thick, i.e.,  $h_1 = 0.6$  mm or 0.7 mm, the percentage contribution of  $\alpha_{d1}$  increases slightly, reaches a maximum and then begins to decrease. For other combinations of the substrate heights, the percent contribution decreases monotonically with increasing  $\epsilon_{r1}$ .

#### IV. CONCLUSION

The formula advanced by Schneider offers an efficient and simple way to accurately compute the dielectric attenuation coefficient for multi-level, multi-conductor structures with relatively low loss. While this formula was considered to be valid only in the quasi-static region,

using a lossless, full-wave formulation for  $\epsilon_{\text{eff}}$  and a finite difference approximation for the partial derivative gives accurate results over a wide range of frequencies, including the dispersive region. In addition, this approach achieves accurate results with computer times up to three times faster than a comparable full-wave, lossy technique. The amount of time that is saved by using this technique is dependent on the total number of dielectric layers as well as the number of layers that are lossy. Another advantage is that this technique can be used to compute the dielectric attenuation coefficient of all the modes in multi-level, multi-conductor structures with significant savings in time. In addition, the nature of the formulation allows it to isolate the contributions of each of the dielectric layers to the total attenuation coefficient, without any additional computations. Thus, the technique presented here offers many advantages for CAD applications; faster computation with very accurate results, applicability to multi-level multi-conductor structures, and the use of existing software.

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